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NSWC TR 87-164

PRIME-RICH ROW EQUATIONS OF THE "SPECIAL" ARRAY ﹐

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BY   R. S. SERY

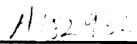RESEARCH AND TECHNOLOGY DEPARTMENT

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In two previous reports, NSWC TR85-120 and NSWC TR 86-516, a method of finding prime-rich equations of the type $I=x^2-x+c$, $c+2N-1$, $N=1,2,3,\ldots$ was described. It was based on an analysis of a certain array, discovered by the author, which has the property that every column of the array can be described by the Diophantine form (integer solutions only) of the above equation. In addition, the rows of the same array can be represented by the related equation $I=x^2+x-r$, $r=2N-1$, and $N=1,2,3,\ldots$ A number of prime-rich "column" equations were found by the method described. This report applies the same method of analysis to show that there are about as many "row" equations (10) which are richer in primes than Euler's equation, $I=x^2-x+41$, as were found for the "column" equations (16).

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FOREWORD

It is of continuing interest to the Navy and the DoD to learn something about the distribution of prime numbers and also to explore methods of devising prime-rich equations. This report is a continuation of work previously done with a view to contributing to the use of *prime number theory* in encription and other fields.

The analysis on which this report is based was done on the employee's own time and it is being published by the Center because of the relevance of the subject to the Navy.

Approved by:

Jack R. Dixon

JACK R. DIXON, Head
Materials Division

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INTRODUCTION

In previous reports[1,2] a special array of the odd positive integers was described for which solutions of the Diophantine equation $I = x^2 - x + c$ corresponded to the columns of the array and solutions of $I = x^2 + x - r$ corresponded to its rows; c and r are each equal to 2N-1 and N=1,2,3,.... represents both the column and row numbers respectively. An infinite set of "primitive cell arrays" (PCA's) was postulated to exist. Each such array is derived from the special array and is limited to the prime values of c (and r) in the equations above. Table 1 shows a section of the original array with prime values of I underlined along with its PCA for c=r=3 shown below it. It was proved[1] that the PCA's for 3 and 5 are infinite in extent and conjectured that they exist for all prime values of c (and r). Based on the properties of these arrays a new method was devised for finding prime-rich "column" equations whose densities, D_c, were comparable to that of Euler's equation, $x^2 - x + 41$ for which, for x=1 to 40, all values of I are prime i.e $D_{41} = 100\%$. The method of analysis involved locating those columns for which no value of I was $\equiv 0 \pmod P$, P is prime, no matter how far the column extended and then finding coincidences of columns "empty" of 3's and 5's i.e. no I's $\equiv 0 \pmod{3,5}$ and, in general, columns where no I's are congruent to $0 \pmod{3,5,7,......,P}$. Equations and columns were used interchangeably as are rows and equations in this paper. Columns or rows are called prime-rich if the density of primes is $\geq 67.5\%$ where the density is defined as $D_c = 100 \times$ (prime values of I)/(total number of I's); the total number of I's is equivalent to x_N and will be considered to be = 40 unless otherwise stated. The columns were treated at some length elsewhere[2], but the rows were mentioned only briefly. However as the results for the rows are just as significant and interesting as are those for the columns, they are presented here. The analysis for the rows is very similar to that developed for the columns and will not be repeated but is described in reference 2.

ANALYSIS

It is clear that for the columns all integers (I's) are positive but that for the rows some values of the I's will be negative. This means that for some prime-rich equations there may be duplications of primes; for example for $I = x^2 + x - 169$ the negative values are -167,-163,-157,-149,-139,-127,-113,-97,-79, -59,-37,-13 and the positive values are 13,41,71,103,137,173 There is one duplication of primes i.e. ±13. The density D_{169} for $x_N = 40$ is 90% (36 out of 40 values are prime) If the negative value, -13 is excluded D_{169} is 87.5%. However as x_N is increased the differences in the two percentages diminishes and, for x_N very large, becomes negligible. For example for $x_N = 2398$ the two

percentages are: 40.8673... if the duplicate is included and
40.8256... if it is excluded. Another instructive example is
$I=x^2+x-7759$. For $x=1$ to 87 the values of I range from -7757 to
-103; for $x=88$ to 124 the corresponding range of positive values
is from +73 to +7741. There are two duplications of integers
namely ±4987 and ±7003. The x values which give the first pair
are 112 and 53 respectively and for the second pair 121 and 27.
However 4897 and 7003 are composite numbers! This means that all
the percentages D_{7759} for x_N = 40,100,200,....,10,000 and beyond
need no correction with respect to the elimination of duplicate
primes no matter how large x_N becomes. The correction factor
required, if any, for Table 4 will be discussed below. The
computer program developed for determining the densities of
primes is the same as that described previously with the
exception that it was modified to record the absolute values of
the negative I's.
    Table 2 shows those equations which describe which rows
contain no integers ≡0(mod 3); which have none ≡ 0(mod 5) etc.
For example rows 1,4,7,10,.....,3N+1 contain no I's ≡ 0(mod 3).
Rows 2,7,12,..... and a second set 5,10,15,.... have no
I's ≡0(mod 5). By solving these equations e.g. $3N_1+1$ and $5N_2+2$
and also $3N_1+1$ and $5N_2+5$ those rows "empty" of I's ≡0(mod 3,5)
can be found; for example the equations (or rows) $I=x^2+x-(30N+13)$
and $I=x^2+x-(30N+19)$ where N=0,1,2,3,... are for for rows devoid
of I's ≡ 0(mod 3,5). This technique can be extended to rows for
which there are no I's ≡ 0(mod 3,5,7,11,...,P) P being prime.
    Table 3 shows the result of carrying out this procedure to
determine all rows in which no I's are ≡ 0(mod 3,5,7,11). All of
the 30 possible combinations are shown in the table and the
results of the solutions for each combination are shown in the
last column. For a comparison with a similar set of "column"
equations see Table 1 of reference 2.
    All 30 row equations were evaluated for $x=1$ to 40 in order
to compare them with Euler's equation for which D_{41} =100%. Next
N was varied from 0 to 500 for all thirty. The program which
incorporated a "Fast Prime Search"[3] program as a subroutine was
designed to select and print out only those rows out of the 15030
examined for which D_r ≥ 67.5%. Eighty five equations which met
this criterion were found. Only 16 of these are listed in Table
4. These are the ones which continued to show high D_r values as
x_N was increased and were better than or compared favorably with
Euler's equation for values of x_N=2398 (where D_{41}=50%). It can
be seen from the table that ten of these equations have higher
D_r's than Euler's for x_N>2398. Figure 1 is a plot for a few of
these "curves" of D_r (x axis) as a function of x_N. Sixteen of
these curves were averaged and the results are also plotted in
the figure. It is a relatively smoothly varying curve as
compared to the individual curves shown.
    With respect to the fact that all rows but the first contain
some negative I's it turns out that the densities D_r for all but
two of the equations of Table 4 are not even affected! A simple

two of the equations of Table 4 are not even affected! A simple
program* was devised to compare those positive and negative
values of I, whose absolute values were identical in order to
determine how many prime pairs existed for a given row. From
Table 4 it can be seen that this occurred rarely. The last
column of the table shows how many duplicate pairs of primes did
occur. Of the sixteen equations only two were found to have
duplicate primes namely r=6163 and r=42,463. The former had one
prime pair and the latter had one prime pair and one composite
pair The entries C1, C2, etc. in the column indicate the number
of composite pairs and those shown as P1 etc. indicate the number
of prime pairs. This means that no corrections for duplications
of primes need be made for any but the two equations mentioned
above and for these the correction is negligible for large x_N and
is easily calculated. In column x_N=10,000 of Table 4 the
correction,if prime duplicates are excluded, is equal to -0.01%!

CONCLUSIONS

The search for prime-rich equations of the type $I=x^2-x+c$ has
been extended to the related equations $I=x^2+x-r$. In neither
case has the search been exhaustive but it is apparent that most
such equations, up to c and r values of ~ 1,500,000, have been
found. The search was limited to those equations whose densities
were >67.5. Of the 85 equations which met this criterion most
can be considered prime-rich but fall somewhat short of the
"standard" chosen, i.e. Euler's equation. Nevertheless they do
merit future study. Similarly there are quite a few equations
not even considered which have densities of 60 to 65% which may
also be worth examining. One example is $I=x^2+x-1,384,907$,
D_r=65%. Its density, for x_N=8000, was found to be 38.725%; its
projected D_r for x_N is about 36%. As mentioned in a previous
report another facet worth examining is that of finding equations
which are very prime-poor.

An additional object of this search has been accomplished
and that was to devise a method of determining which of a large
number of equations might be prime-rich. It was done by finding,
by use of the "Special" array and its derived primitive cell
arrays, those columns and rows (corresponding to the "c" and "r"
equations) which contained no integers $\equiv$ 0(mod 3,5,7,.....,P)
where P was limited, for practical reasons, to the prime values
3,5,7,11.

Finally is is felt that the work described above has,
perhaps, contributed a bit of an insight into the somewhat
organized randomness characteristic of the distribution of prime
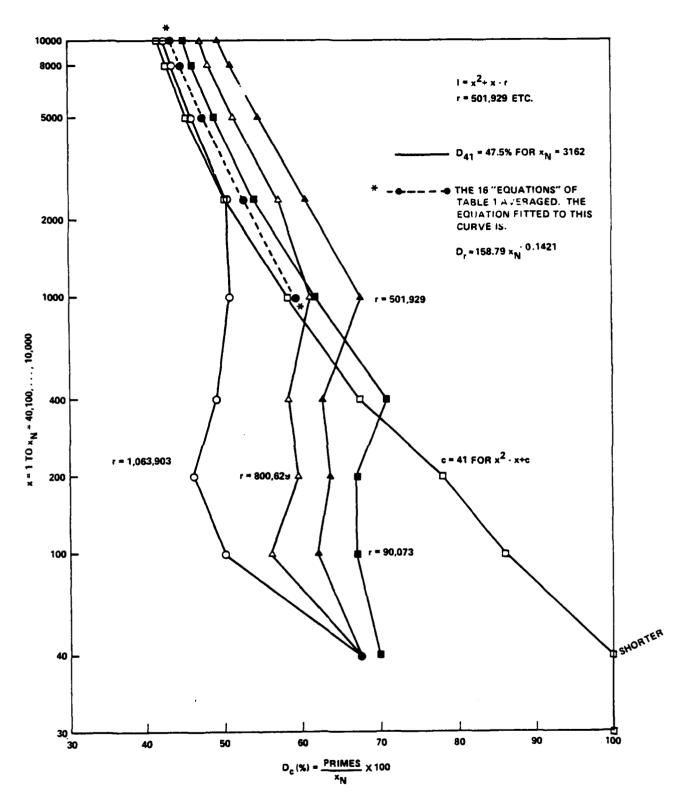numbers.

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*
see Appendix A

FIGURE 1. SEMILOG PLOTS OF D_c VS x_N FOR 4 VALUES OF C FROM TABLE 1. THE SIXTH, SHORETR, CURVE IS AN AVERAGE FOR ALL 16 SETS OF DATA OF THE TABLE (DATA FOR c = 41 EXCLUDED)

TABLE 1.   "SPECIAL" ARRAY OF ODD INTEGERS WITH PRIMES UNDERLINED
WITH THE DERIVED PRIMITIVE CELL ARRAY OF 3 SHOWN BELOW IT

$$I_c = x^2 - x + c$$

$c=1$	3	5	7	9	11	13	15	17	19
$x_1=1$	2	3	4	5	6	7	8	9	10

$r=$

1	1	5	11	19	29	41	55	71	89	109
3	3	9	17	27	39	53	69	87	107	129
5	7	15	25	37	51	67	85	105	127	151
7	13	23	35	49	65	83	103	125	149	175
9	21	33	47	63	81	101	123	147	173	201
11	31	45	61	79	99	121	145	171	199	229
13	43	59	77	97	119	143	169	197	227	259
15	57	75	95	117	141	167	195	225	257	291
17	73	93	115	139	165	193	223	255	289	325
19	91	113	137	163	191	221	253	287	323	361
21	111	135	161	189	219	251	285	321	359	399
23	133	159	187	217	249	283	319	357	397	439
25	157	185	215	247	281	317	355	395	437	481

Primitive cell array for $c = p = 3$*

```
 -   -   -   -   -   -   -   -   -   -
 3   3   -   3   3   -   3   3   -   3
 -   3   -   -   3   -   -   3   -   -
 -   -   -   -   -   -   -   -   -   -
 3   3   -   3   3   -   3   3   -   3
 -   3   -   -   3   -   -   3   -   -
 -   -   -   -   -   -   -   -   -   -
 3   3   -   3   3   -   3   3   -   3
 -   3   -   -   3   -   -   3   -   -
 -   -   -   -   -   -   -   -   -   -
 3   3   -   3   3   -   3   3   -   3
 -   3   -   -   3   -   -   3   -   -
 -   -   -   -   -   -   -   -   -   -
```

*   In the primitive cell array (PCA) for every integer in the
"Special" array not divisible by 3 the corresponding position in
the PCA is left blank whereas each integer $\equiv 0 \pmod 3$ is replaced
by the numeral 3.

TABLE 2. EQUATIONS SHOWING THE NUMBER OF SETS OF EMPTY ROWS FOR
EACH PRIME VALUE OF P FOR THE PRIMITIVE CELL ARRAYS

P	No. of sets	Sets of "empty" rows per array (N=0,1,2,..)
3	1	$3N+1$ *
5	2	$5N+2, 5N+5$
7	3	$7N+1, 7N+2, 7N+6$
11	5	$11N+2, 11N+3, 11N+4, 11N+8, 11N+11$
13	6	$13N+2, 13N+3, 11N+5, 13N+6, 13N+11, 13N+12$
17	8	$17N+1, 17N+4, 17N+5, 17N+6, 17N+8, 17N+14, 17N+16 \ 17N+17$
19	9	$19N+2,3,4,5,7,9,14,15,18$
23	11	$23N+1,3,5,6,7,8,14,16,19,20,23$
29	14	$29N+3,5,8,9,10,11,13,17,20,24,26,27,28,29$
31	15	$31N+2,3,4,5,7,8,10,11,15,18,23,24,25,27,29$
37	18	$37N+1,2,4,6,8,9,12,14,15,16,17,21,26,30,31,32,33,35$
-	-	-
-	-	-
P	$(P-1)/2$	$PN+M_1,M_2,M_3,\ldots\ldots\ldots\ldots,M_{(P-1)/2}$

*Subscripts such as 1,2,3, etc. in $3N_3+1$, $5N_2+2,\ldots\ldots$ as shown in Table 3 and
elswhere were omitted for the sake of clarity

TABLE 3.   COINCIDENCES OF EMPTY ROWS FOR DIVISORS 3 AND 5; AND 3,5,7; AND 3,5,7,11 FOR DERIVED PRIMITIVE CELL ARRAYS

No. of K family	Equations for "empty" rows for:				Families of equations of form x^2+x-r: $r=$
1	$3N_1+1$	$5N_2+2$	$7N_3+1$	$11N_4+2$	$2310N+$ 883
2	"	"	"	" 3	" " 1303
3	"	"	"	" 4	" " 1723
4	"	"	"	" 8	" " 1093
5	"	"	"	" 11	" " 43
6	"	"	$7N_3+2$	$11N_4+2$	" " 1543
7	"	"	"	" 3	" " 1963
8	"	"	"	" 4	" " 73
9	"	"	"	" 8	" " 1753
10	"	"	"	" 11	" " 703
11	"	"	$7N_3+6$	$11N_4+2$	" " 1873
12	"	"	"	" 3	" " 2293
13	"	"	"	" 4	" " 403
14	"	"	"	" 8	" " 2083
15	"	"	"	" 11	" " 1033
16	"	$5N_2+5$	$7N_3+1$	$11N_4+2$	" " 2269
17	"	"	"	" 3	" " 379
18	"	"	"	" 4	" " 799
19	"	"	"	" 8	" " 169
20	"	"	"	" 11	" " 1429
21	"	"	$7N_3+2$	$11N_4+2$	" " 619
22	"	"	"	" 3	" " 1039
23	"	"	"	" 4	" " 1459
24	"	"	"	" 8	" " 829
25	"	"	"	" 11	" " 2089
26	"	"	$7N_3+6$	$11N_4+2$	" " 949
27	"	"	"	" 3	" " 1369
28	"	"	"	" 4	" " 1789
29	"	"	"	8	" " 1159
30	"	"	"	11	" " 109

TABLE 4. EQUATIONS x^2+x-r FOR r=501229, etc. WHERE THE DENSITY, D_r IN PERCENT (100xPRIMES/40) FOR x=1 TO 40 IS > 67.5% AND WHERE D_r IS ALSO GIVEN FOR x_N =100, 200, ETC.

Equations having higher D_r values then c=41* for x_N= 10,000

	x_N=40	100	200	400	1000	2398	5000	8000	10000	±Pairs
501229	67.5	62	63.5	62.5	67.5	60.34..	54.3	50.625	49.250	C1
349513	67.5	68	65	65.75	68	59.96..	53.62	50.275	48.85	0
445473	67.5	63	59.5	62.5	65.9	58.84..	52.68	49.65	47.95	C1
800629	67.5	56	59.5	58.25	61.1	56.96..	51.16	47.8375	46.87	0
249439	72.5	67	64	64.75	65.5	56.79..	51.06	48.1	46.65	0
82009	72.5	65	64.5	68.75	60	51.41..	46.8	44.475	43.32	C1
90073	70	67	67	71	61.5	53.79..	48.82	44.8875	44.7	C1
152839	75	59	61.5	65.5	60.8	52.87..	47.74.	45.05	43.71	0
366349	67.5	56	52.5	54.25	57.3	51.79..	47.0	43.625	42.19	0
1063903	67.5	50	46	49	50.7	50.25..	45.84	43.325	42.1	C2
41*	100	86	78	67.5	58.1	50	45.22	45.575	41.49	-

Equations with D_r values close to those for r=41

	x_N=40	100	200	400	1000	2398	5000	8000	10000	±Pairs
53509	75	63	66.5	68.5	56.3	49.58..	43.68.	41.325	40.11	C2
7759	67.5	77	70.5	62.5	54.5	47.83..	43.18.	40.525	39.25	C2
2293	77.5	77	68.5	63	54.5	47.28..	42.94.	40.3625	39.25	P1
6163	70	75	67.5	61	53.8	48.12..	43.1	40.375	39.16	C1,P1
23083	67.5	67	71.5	63.75	54.3	47.70..	42.8	40.375	39.15	P1
42463	70	60	63	61.5	54	47.83..	42.68.	39.95	38.94	C2

*Euler's equation, x^2-x+41, a "column" equation, is included here for the sake of comparison with the row equations.

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1. Sery, R. S., Relationships Between Prime-Rich Euler Type Equations and a Triangular Array of the Odd Integers, to be published in the Journal of Recreational Mathematics (also appeared as NSWC TR 85-120).

2. Sery, R. S., A New Method for Finding Prime-Rich Equations of the Type X^2-X+C, NSWC TR 86-516, Dec 1986.

3. Varsano, S., EDN, (also personal communication), Jan 1985, p. 246.

Appendix A

If only odd integer solutions of the equation $I=x^2+x-r$ are considered then it is obvious that x and r <u>must</u> be integers. If there exists a pair I_1, I_2 such that $I_1=-I_2$ then 2 equations exist i.e. $I_1=x_1^2+x_1-r$ and $I_2=x_2^2+x-r$. Adding the two together yields:

1. $\quad x_1^2+x_1+(x_2^2+x_2-2r)=0$

and solving for x_1,

2. $\quad x_1=1/2(-1+ \sqrt{\{8r+1-4x_2[x_2+1]\}})$

where the minus value of the square root sign is ignored as all x_1, x_2 values are positive.

For a pair of integers x_1, x_2 to exist such that $I_1=-I_2$ the right hand side of equation 2. must be an integer i.e. the expression in parentheses is even and that within the square root sign is an odd square. If x_1 is not an integer for any value between 1 and $\sqrt{r}$ then there are no duplicate I's whether prime or composite.

Another program beside the one shown below was used to search for duplicate I's. It involved examining only those values of x_1 between $\sqrt{r}$ and $\sqrt{(2r)}$ as there are fewer positive values of I in this range than in the corresponding range of negative I's in the range x=1 to $\sqrt{r}$. Both programs were designed to indicate and print out only those matching pairs of I's (and pertinent x values) which were found. The somewhat simpler but slighly longer program shown below was used to confirm the results of the one mentioned above

The Basic program run on the HP 9836C

```
10     ! This is primedupl 1, of 3/9/87
20     OPTION BASE 1
30     DIM Kr(16)
40     DATA 2293,6163,............,1063903 *
50     READ Kr(*)
60     FOR Rr=1 TO 16 STEP 1
70     Rp=Kr(Rr)
80     Sr=SQR(Rp)
90     Si=INT(Sr)
100    IF (Sr-Si)>.5 THEN
110    Sr=Sr+1
120    END IF
130    FOR X=1 TO Sr-1 STEP 1
140    P=SQR(8*Rp+1-4*X*(X+1))
150    Pi=INT(P)
160    Ps=P-Pi
170    IF Ps>0 THEN GOTO 200
180    IF Ps=0 then
190    PRINT "P=";P;"X=";X;"THERE IS A PAIR +P,-P FOR Rp";"Rp=";Rp
191    END IF
200    NEXT X
201    NEXT Rr
210    END
```

* See the first column of Table 3 (value 41 not included in 40 above)

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